

# On the objective existence of physical processes: a physicist's philosophical view

J. Sanchez-Guillen and R.A. Vázquez

*Departamento de Física de Partículas, Campus Sur*

*15782 Santiago de Compostela, Spain*

## Abstract

Inspired by philosophical ideas of Boltzmann, which are briefly recalled, we provide strong support for the possibility and convenience of a realistic world picture, properly nuanced. The arguments have consequences for the interpretation of quantum mechanics, and for relevant concepts of quantum field and string theory, like monopoles and branes. Our view is illustrated with a cybernetic analogy and complemented with a summary of the basic philosophical concepts.

## I. INTRODUCTION

It is common in the Physics literature to find more or less involved statements on the existence of the basic objects and of their representations. Relevant examples today may be strings or branes, as atoms and quarks were in the past. As a practical or more conscious attitude, the positions range from two extremes. On one side, existence is granted or denied to the items if they can be detected (like atoms or quarks) or disproved, like the ether or the planet Vulcan, without further thoughts about the reality of their mathematical representative. In the other extreme [1] existence is assigned even to the representations themselves, with varying degrees of commitment to the philosophical concepts.

In a way this parallels the extremes of the oldest philosophical debate, between rationalists and empiricists, the most radical forms of which are naive realism and constructivism. Practitioners of physics are usually closer to the former view. Nevertheless one cannot always ignore the philosophical background, and certainly not concerning the foundations of quantum mechanics, where the interpretation, which is a basic ingredient, has always been a matter of debate, often called the problem of realism. This debate can have also practical implications e.g. for quantum information or quantum cosmology. In any case, thinking about these abstract matters certainly opens our vision and improves our questions and understanding.

Our purpose is to provide a clear and simple conceptual (philosophical) picture of these important questions, which could also be useful in practice, clean of technicalities. Our view was mainly built thinking about the philosophical ideas of Boltzmann. They are contained in his only specific technical article [2], which is both long and hard to read, in his notes for the Philosophy lectures held by Boltzmann in his last years [3], which are even harder, and also informally scattered in his Popular Writings and other popularizations, which are very rewarding [4].

In the next section II we address the main problem of the existence of the physical world and its description, beginning with a historical perspective to provide a selection of Boltzmann's ideas, and we reassure the reader immediately with the basic conclusion, that one can safely maintain a realistic position on the objective existence of the external world, in permanent evolution, adjusted and regulated by experimental confrontation. In Boltzmann's words "*we must adopt the objective point of view*", as the phrase goes.[2]. This will lead

us to our main proposal, to call existing or non existing only what is detectable or falsable respectively. As for the different concepts, at various levels of complexity and abstraction, for which neither option can be achieved at present, one should distinguish between the physics approach with mathematical analysis and measurements, and the philosophical reasoning, involving qualitative attributes which can be related to existence. Interesting examples of the former are entanglement or virtualities, and of the later unavoidability, which is discussed in Section III. There we apply these ideas to specific concepts of quantum field theory, string theory and cosmology, like monopoles and branes and to the general problem of the interpretation of quantum mechanics. Section IV summarizes the conclusions and outlook. In Appendix 1 we illustrate the mental representations with a model analogy from neural networks and programming languages. We include another Appendix 2 to explain in a concise way the philosophical background, relevant for our arguments and beyond.

## II. THE GENERAL PROBLEM OF THE EXISTENCE OF THE EXTERNAL WORLD

To put the problem in a useful historical perspective, we remind that Boltzmann, who had based the (mechanical) understanding of thermodynamics on atoms, had to discuss and defend them frequently against the radical philosophical positivism as well as against extreme phenomenological theorists. The present work is in part an attempt to make more accessible his ideas and to start applying them to present physics. The first point of Boltzmann Philosophy was the necessity to define clearly the concepts discussed and in fact this is how his purely philosophical work begins and ends, illustrated with examples and personal anecdotes. This claim for clarity is a constant in his writings, urging to prevent the perverse antinomies of philosophy. This will be present in the discussion on the *existence* now and we shall keep it in mind in the applications below, especially to the interpretation of quantum mechanics.

In his article Boltzmann goes immediately after this introduction to describe the process of human perception of the external world, starting from the observation[22] that *the laws according to which our own perceptions run their course are familiar to us and lie ready in the memory. By attaching these same pictures also to the perceptual complexes that define the bodies of others, we obtain the simplest description of these complexes.* This is elaborated

further arguing that in the extreme idealistic position *the sensations and volitions of all others could not be on the same level as the sensations of the observer, but would have to be taken as merely expressing the equations between his own sensations*. The idea is stated more clearly in the next page: *Therefore we designate these alien perceptions with analogous mental signs and words to those for our own, because this gives a good picture of the course of many complexes of sensations and this simplifies the world picture*. For clarity and economy, Boltzmann claims after another couple of pages that *we must adopt the “objective point of view”, as the phrase goes. It turns out that the concepts we linked with “existing” and “non existing” largely remain applicable unchanged. Those people or inanimate things that I merely imagine without being forced to do so by the regularities in complexes of perceptions do not exist for others, they are “objectively” non-existing*.

The main conclusion of this line of thought, based largely on the *common judgement of all*, implies in (our) in simple terms, that one can maintain a realistic position, assuming the objective existence of the physical world, in a reasonable degree of agreement with our representations thereof, which are sufficiently universal and which may evolve as required by the experimental confrontation and of course by our own evolution, an essential ingredient of Boltzmann Philosophy. For instance he wrote: *The brain we view as the apparatus or organ for producing world views, an organ which because of the great utility of these views for the preservation of the species has, conformably, with Darwin’s theory, developed in a man to a degree of particular perfection just as the neck in the giraffe and the bill in the stork have developed to an unusual length [2]*.

Let us explain briefly the argument, which being *philosophical*, has to use logic starting, of course, from our mental representations, the concepts. We recognize them and decide whether they are relevant or not from confrontation with the representations of others. But those are also external *objects*, so that any statement and mental construction is actually based on the external world, as represented with enough degree of fidelity and universality by our concepts. The adequacy or correspondence of the reasonably universal representations (concepts) is based on confrontation and guaranteed by evolution, which also renders the process dynamical (over large time scales). Therefore it would be *inconsistent* to *deny* the reality of the external world. Notice that one has not *proved* its existence, but the absurdity of the attempts to disprove it, thereby establishing the possibility and convenience of an objective world picture.

Boltzmann uses then an ingenious argument to maintain this universal realism for any kind of brain process and beyond, applying these reasoning successively to simpler and simpler organisms, reaching the virus and molecular levels, until confrontation or detection is generalized to interactions. Our starting point, the mental processes, beginning with the primary physical inputs, which are later elaborated in different degrees of robustness and complexity, yielding the representations with varying degrees of fidelity and universality, should become ultimately also a question of bio-physical interactions. But for our philosophical arguments suffice to say in that respect that the agreement between nature and reason is because reason is natural and not the opposite. In Appendix1 we provide a cybernetic analogy of the cognitive process, which can be useful.

In any case, from the arguments so far, i.e. from clear conceptions, rigour and logic, one has established the possibility of an "objective" world view, and the convenience of this representation, provided it is sufficiently contrasted and updated. This point of view can be seen a golden middle between the two extreme positions, as illustrated in Appendix2.

This leads further to propose, that we call *existent* only those representations which, clearly defined, are physically realizable and detectable (in principle with some energy transfer [5]). A consequence is that there is no place for gradations in this clear, but restricted notion of existence: representations which fulfil it correspond to existing objects, like atoms or neutron stars, while those which do not, should not be called existing. This notion of existence has many advantages, like a highest degree of universality. *The assumption of different degrees of existence would be decidedly inappropriate* as Boltzmann says, and *the denotation must always be so chosen that we can operate with the same concepts in the same way under all circumstances, just as a mathematician defines negative or fractional exponents in such a way that he can operate with them as with integral ones* [23]. This avoids confusions, like most of the dreadful antinomies of philosophy, but it also rises the following problem.

There are concepts which can be clearly disproved, like ether or the planet Vulcan. But for many ideas one is not able, at least for different time periods, to detect or disprove them as defined. What can then be said about such useful representations, with respect to the external world? We have to address the problem, because as we said, any statement of any kind involves the representations.

This marks a line between a physics approach, where one has to look for a verification or

falsification, and the philosophical, where one can envisage attributes, which can be related to existence more or less directly on the one hand, but which have the possibility of qualitative, more or less coarse gradations, on the other. They can be vague, like clarity, simplicity or beauty, or very sharp, like *(in)dispensability*, which is discussed in the next section at length because of his potential relevance.

A first consequence of this is the great convenience of distinguishing in physics between qualitative concepts, but which are ultimately philosophical interpretations, from genuine physical proposals, always falsable in a way or another. Textbook examples are many of the different proposals to render quantum mechanics *complete* or more understandable, which will be discussed in III B. There is of course place in Physics for useful qualitative discussions, even in our restrictive philosophical view, as we discuss next.

### III. A PHILOSOPHICAL FRAMEWORK FOR THE EXISTENCE IN PHYSICS

#### A. The unavoidable attributes related to existence

One of the simplest and useful examples of such general predicates related to existence can be *indispensability* ( or the closely related concept of *unavoidability*), introduced by Boltzmann less formally for the concept of the atom in his popular writings. They are not present in his technical technical publication, and so their analysis below, is (needless to say!), essentially ours.

Boltzmann argued in his talks and popular writings, that his ideal atoms were, not only useful, but indispensable. They became of course properly existing after Einstein computed (following Boltzmann's prescription for fluctuations) the observed Brownian motion of pollen and made predictions confirmed later by Perrin [6]. At this level the concept of atoms was defined simply as elementary grains of matter. Of course concepts have to be defined precisely, and that of atom was finely sharpened later. More specifically, they are represented by complex functions, solutions of linear differential equation (Schrödinger's), which in turn can be combined, enlarging at will their potential manifestations, as discussed in III B.

To illustrate this further let us use the concept of gen, similar in some ways to the atom. With a very general definition, as units of transmittable information, one can of course call them existing, after their molecular structure was found in 1953, but in hindsight one could

have shown them to be unavoidable, at least since W. Sutton named them in 1922 as the Mendelian units of transmission. Life is even more difficult to define than gen, but we think it should not be difficult to show it to be unavoidable under rather general conditions. As for conscious life, it seems to us almost hopeless at present to define and accordingly much more difficult to argue its unavoidability, as Drake “equation” shows.

Notice the subtle difference between indispensability (Unentbehrlichkeit), which refers to the object  $\rightarrow$  concept direction, the “easy way” according to Boltzmann, and avoidable (Vermeidlich in German or evitabile in Latin languages) more related to the concept  $\rightarrow$  object direction, which is harder the more complex the concept, as is clear and it is illustrated in Appendix2. This admittedly exaggerated subtlety, can nevertheless illustrate the impossibility of a perfect one to one correspondence between concepts and the external world, as we shall see is required by some attempts to make quantum mechanics “complete”.

Still in biology let us remind of another example given by Boltzmann for non existent concepts in the philosophical article: the unicorn. It turns out that today one can speak of the realizability of such a concept, and in fact it has been done already, e.g. with drosophila flies. It seems on the other hand unlikely to be neither unavoidable from evolution, which is difficult anyway, nor at least stable under it. This can illustrate the role of evolution in the notion of existence as stated above, and of the relevance of dispensability.

Another interesting example, back to physics, is the concept of the electromagnetic field, (missed by Boltzmann). Of course the concept of electric *and* magnetic fields, became almost unavoidable after Hertz discovery, and certainly with the disproof of ether. But after the success of quantum gauge theory the concept of electromagnetic field, *the vector potential*, is clearly unavoidable. This shows the new level one reaches when a concept is defined in terms of mathematically deeper theories, as the fundamental interactions and constituents.

Our next discussion involves in fact concepts, monopoles and branes, which are expressed mathematically, with higher degrees of abstraction and complexity. Of course the first question in physics is whether the objects can be realized, or detected, i.e. registered in processes involving some energy transfer and which can be reproduced. The properly defined concept will correspond in that case, and only in that case, to an existing object, or, shortly, *exists* (in the sense of the definition). Until this can be achieved one can discuss questions as how fundamental or *effective* is the corresponding mathematical theory, but one can also make progress from a more philosophical point of view as the analysis of the attribute of

indispensability, applied to the following relevant examples shows.

The concept of monopoles corresponds to the sources of the magnetic field, i.e. magnetic charges. They were shown by Dirac to be a way of implementing discreteness of the electric charge, which requires regularization of singularities and conservation of symmetries. Its existence has not been proven so far, although there is a more recent claim in condensed matter physics experiments, with a special composite called *spin ice* [7]. It is a matter of debate at present as to whether these objects fulfil the requirements of the general class required for discrete charges. On the other hand there are arguments in favour of the indispensability of monopoles: fundamental theories with compact (gauge) phase symmetries, the so called grand unified, imply trivial discreteness of charge, but at the same time, they also predict monopoles. Grand Unification of interactions could be established soon. In fact neutrino masses provide a good hint. This example also shows the importance to consider, as mentioned above, how fundamental is an object, distinguishing monopoles as extended solutions of a fundamental theory from those aggregates of particles combined in atoms, molecules, and further structures.

As the name indicates, the concept of brane [8] refers to extended objects in spatial and temporal directions, introduced or appearing in some string and gravity theories. They are useful for combining gauge fields and gravity at the quantum level, at least in some approximations, and in cosmology. They serve so far an auxiliary role. To decide about their existence one has to consider their energy (or density) and propagation. In fact they provide a way to implement energy conservation for the strings, which mediate their interactions, to make particles or even the universe. Direct observable consequences, to decide if they exist (in a specified class) are very difficult (for instance, there have been proposals for special gravitational waves). Alternatively, one could consider if they are unavoidable from their ability to change the rate of expansion, at present and in the past (inflation). But these are not so well understood [9]. So, in contrast to the case of monopoles, we could have to wait very long to decide about the existence of branes and even about their indispensability. Therefore, it would be convenient on occasion to keep this in mind speaking, or writing about these most interesting concepts. Another conclusion of this section is that although indispensability or unavoidability are at another level (philosophical) than the physical existence, which requires experimental confirmation or falsification, they can be useful even in physics. Besides, they are more flexible and admit with full right loose gradations as *almost*



or the celebrated *for practical purposes*. From a formal point of view, they could be seen as a much weaker form than the mathematical attribute *necessary*, as is appropriate in Physics where experiments ultimately decide.

## B. Interpretation of Quantum mechanics

In quantum mechanics one incorporates the representation from the beginning which may be one of the reasons for its astonishing performances. One works in fact at the level of representations without actual reference to the external world until measurement. These representations are complex "wave" functions, which can be superposed with the interference properties of waves. Our proposal requires not to call these representations *existing* until they have been realized in a detectable way, with a probability given of course by the norm of the combined function. This way one keeps a universal meaning of the concept and avoids the potential confusion of many interpretations which have been proposed to cope with the apparent conceptual difficulties of quantum mechanics. These are mainly the essential probabilistic nature of the description ("stochastic unit samples"), the mechanism and the nature of the transition from an uncertain or fluctuating state to the robust and certain measurement result ("collapse and decoherence"). Interpretations which in a more or less subtle way attribute existence to the representations, the wave functions and their combinations, or to additional auxiliary functions, are explained in many excellent textbooks [10]. Well known examples are the many worlds of Everett, Hartle's consistent histories and the pilot waves of Bohm. It is important to remember there is no way to detect or falsify those interpretations, so we are in fact at the philosophical level, where the relevant question should be whether these auxiliary objects, worlds or paths, are unavoidable. The answer is clearly they are not, rather the opposite, a view shared by many of the active researchers using those fundamental aspects of quantum mechanics, like A. Zeilinger [11]. In fact, it is natural to accept limitations to common sense imposed by physics, as it happened before with simultaneity or with indistinguishability.

This illustrates the usefulness of our simple proposal, but we do not claim that the situation is completely satisfactory. The question whether quantum mechanics is a complete theory is alive since Einstein, Podolsky and Rosen posed it [12]. One has to define completeness, and of course they did: *every element of physical reality should have a counterpart in*

*the physical theory*. This strong requirement is very difficult to meet, as we have seen from the discussion on the cognitive process. In fact, trying to complete the theory to meet the above difficulties one has to cope with very restrictive theorems about the implications of adding new "hidden" variables [13], under very reasonable general conditions, which have been always experimentally confirmed in favour of pure quantum mechanics. There are even stronger theorems limiting the possibility of such extension from internal consistency [14].

There are partial but promising solutions to the last of the basic mentioned problems, decoherence [15], but it is clearly beyond the scope and space limits of this article to enter into details of these well defined physics. Let us remind instead that there are in fact other reasons for insatisfaction beyond these conceptual problems, especially in the relativistic extensions, where infinities appear in the perturbative solutions. These divergences are under control, and some even well understood, which is not the case for the extension to gravitational interactions. Also there are many basic problems related to strong interactions defying solution for decades, like the confinement of quarks. This is seen by some as a need to reformulate the foundations, with new physics [10, 16], which would be interesting to study in our framework. From the simple concept of existence point of view, there are well defined physics notions to measure virtualities and quantum entanglements. Let us comment briefly on a special one [17]. It is based on the worldline or proper time formulation of quantum mechanics, due to Fock, Schwinger, and Feynman. There, one parameterizes quantum amplitudes with an auxiliary parameter called proper time, which controls the fluctuations of the quantum state in spacetime, and are represented as a path integral in that auxiliary space. After some manipulations, the integrals can be computed by combination of numerical and analytical methods, and the fluctuations visualized. The virtuality of the process is related to different random walks, more or less directed, which in turn can be put in correspondence with a Hausdorff dimension, 2 in the extreme quantum or 1 in the the classical regime. This is to illustrate that there are concepts, like virtualities, which can be further analysed with our philosophy proposals and which could help understanding and even visualizing the physics problem.

## IV. CONCLUSIONS

Building on Boltzmann philosophical ideas, briefly recalled, we have given arguments based on general concepts and logic, strongly supporting the possibility and the convenience of maintaining the objective existence of physics processes. They imply in turn to *restrict* that *existence* to concepts which can have a clear manifestation. For concepts which one is not able for the moment to submit to such a requirement, the philosophical analysis can make useful contributions, with attributes related to existence. We elaborated the simple examples of unavoidability and its reverse. They were first illustrated with historical interesting examples of atoms, fields, and even gens and life and then applied in some detail to relevant concepts at present like monopoles and branes. In quantum mechanics, they provide a clarification and criticism of popular interpretations. More involved analysis have been suggested for future research, in a fruitful interplay of physics and philosophy. We hope that our presentation is not perceived as an over-simplification, a possibility we assumed in the spirit of the practical and direct form Boltzmann declared indispensable for philosophical argumentations.

### Acknowledgements

JSG has benefitted from many conversations with D. Flamm, philosopher, physicist and grandson of Boltzmann and with J. Ordoñez, who provided many of the original references. We were encouraged by the late Julio Abad in previous stages of this work, which will appear in a special volume in his memory. This was supported in part by Ministerio de Ciencia e Innovacion and Conselleria de Educacion under Grants FPA2008-01177 and 2006/51.

## V. APPENDIX1: BASIC PICTURE OF THOUGHT WITH CYBERNETICS ANALOGY

Our arguments started from the basics of the cognitive process, where a physical input triggers a primary signal, which is later processed into a memory or commemorative recordings of various degrees of complexity, the concepts, involving neural networks in specific areas in the brain. They are decanted through permanent confrontation with the external world and those of others, as explained, and become ideas at various degrees of abstraction

and distance from the primary trigger [18]. This basic conception (and logic) is what has been used for the argument supporting an objective world picture, in which those representations, the concepts, can be appropriately said to correspond to existing objects, provided they are detected, or not, if experimentally disproved.

The human brain produces general frames abstracting relevant properties of objects, the languages, consisting of words, syntax, and meaning, of different levels of abstraction and, correspondingly, of retroprojectivity to the external objects. The main areas and processes involved are becoming known, but of course there is a long way to go to master the codes, and it is even an open question whether the whole process can be controlled in a relevant way. Besides, we are aware of the key role of chemistry in the brains functions! But for our philosophical reasoning here we only need the general concept, which can be illustrated with a neural network analogy.

In neural networks, one has a first layer of devices (neurons) receiving an input. This in turn triggers an output from different numbers of neurons to the next layers, with different weights. Combining outputs, simple functions (say  $\tanh$ ) connecting two values (on-off) one can produce complicated functions, which will implement effective operations, like recognising voices or pictures. In terms of computers these simple functions could correspond to *machine languages*. But one has also *object oriented languages*, like C++, where one works with abstract functions (for instance, templates), and can operate with them. In our analogy, these correspond to more abstract and general languages, the most universal of which is mathematics.

Let us look at the concept of *pair*. In the first case, in machine related language, one has pairs of concrete objects (integers, for example), which allow in turn for operations like permutations or orderings. In the second more advanced case, templates provide abstract pairs which can in turn be paired or otherwise operated successively, including combinations with other objects. In the neural network this could correspond to nonplanar and transverse connections, which, require a much larger size and plasticity, like in parallel vector networks, as indicated schematically in Fig.1

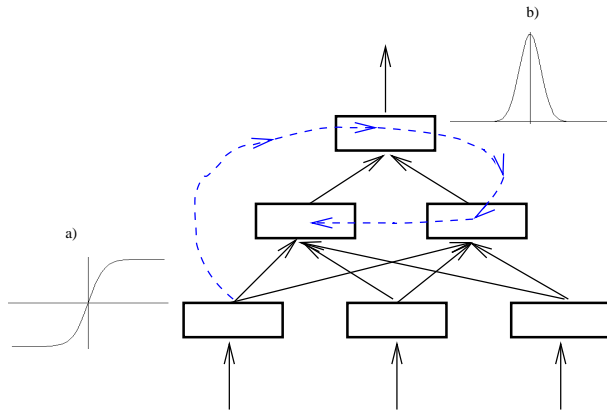


FIG. 1: Schematic representation of a neural network. The input passing through simple functions like (a) is transformed into the output (b). In blue (dashed lines) a possible transversal and reverse connection is shown.

## VI. APPENDIX2: PHILOSOPHY BACKGROUND

As our main conclusion can be seen as a compromise between empiricism and idealism, we explain these basic philosophical ideas and terminology, condensed and simplified, using physical examples.

Kant is a standard reference [20] and for good reason, as mathematics and physics were his starting point, and it is the first manifestly critical approach to the theory of knowledge. His position was the first reasonably, partially realistic one. In the old debate of empiricism, denying reality of ideas and the external world and rationalism, assigning it to both of them, Kant's proposal is a middle solution: he did not grant general existence to ideas but he defended some existence of the sensorial world. Boltzmann's position can be seen as a big step forward in this direction, with a sound scientific basis, including evolution and opening it to future progress.

Back in 1781 [21], Kant noticed that statements can be *analytical* (e.g. the electrons with up or down spin directions in a factorized tensor product state) or *synthetical* (e.g. the electrons in a symmetrized coupled (entangled) state with a given total spin, 0 or 1 ). More schematically: “ $a$  is in  $ab$ ” is analytical while “ $c$  is in  $ab$ ” is synthetical.

All empirical (called a posteriori) statements are synthetical (experience always teaches), but the opposite is not true, not all synthetic statements are empirical: there are some statements with new properties in addition to the premises (synthetic) which are true in-

dependent of empirical experience (a priori). This simple scheme was thought to apply to mathematics and even some concepts of physics but it has been generalized (and relativized) by modern Philosophy of Science.

A proper analysis is also the task of neuroscience and the theory of knowledge as discussed, but as a philosopher Kant argued that there are preconditions in humans, such as time and space, which universally allow such processes. Of course Kant, a devoted Newtonian who had himself worked out the notion of galaxies, took time and space as universal and absolute, as well as other necessary ingredients of thought called *categories*. They are fundamental ideas like *causality*, which had been around from the beginning of Philosophy and ordered by Aristotle. Needless to say that those absolute notions were naive, and wrong in strict terms, but the framework was adequate for scientific discussions and the seed for the later developments. It is worth reminding ourselves that Boltzmann who proposed a big step forward, facing the ontological problem [3], warned against absolute use of “laws of thought” like causality, *which we may denote either a precondition of all experience or as itself an experience we have in conjunction with every other experience*[2]. He also warned frequently that in the realm of explanations, models and theories could be also useful, even if apparently wrong.

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